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Application Note AN-01 (Revision A)

Extending the Frequency Response of the MHD Angular Rate Sensor

The Magnetohydrodynamic Angular Rate Sensor (MHD ARS) has a very wide bandwidth when compared to other types of rate sensors. The MHD ARS is designed to measure angular motion over a 1 Hz to 1,000 Hz frequency range. Figure 1 provides the magnitude responses for several of ATA Sensors' transducers and demonstrates the large bandwidth capability of the MHD ARS. Exhibiting a wide rate bandwidth is ideal for many applications, including crash dummy testing, active control of pointing systems, and measuring the rotational vibration of complex structures. However, engineers interested in other applications, such as biodynamic, ergonomic, vehicle roll-over, and short duration inertial navigation are interested in the lower frequency end of the ARS measurement spectrum, typically below 10 Hz.

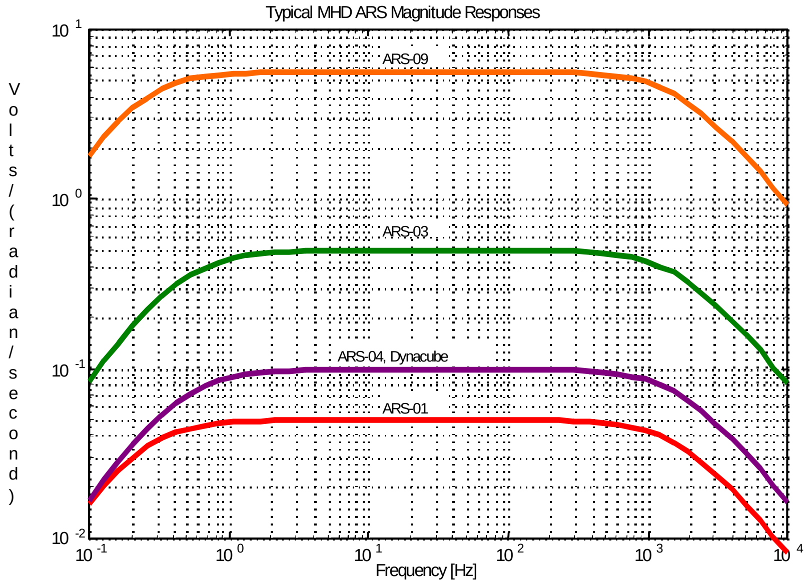

Figure 1. Typical Frequency Responses for Several Magnetohydrodynamic Angular Rate Sensors.

If attitude or angular position knowledge is required for longer periods of time than a few hundred milliseconds, then it is desirable to extend the effective low frequency corner of the MHD ARS such that the rate or displacement error is minimized. A recursive digital filter can be implemented to perform the frequency extension. The advantage of using a digital filter as opposed to an analog filter is that the digital filter can be conveniently reconfigured, even updated adaptively during operation whereas an analog filter requires additional board space and power and cannot be easily reconfigured for varying applications.

Extending the bandwidth at the lower frequencies is viable for time histories of only a few seconds in most cases. This is because the sensitivity of the MHD ARS will fall off at least 20 dB per decade as the frequency approaches zero Hz (steady state or constant rate). In other words, the MHD ARS cannot measure a steady state rate like a gyro and will have a zero output with a constant rate input. However, on the other end of the spectrum, there are very few rate sensors that can measure angular rate above 1 kHz. Most gyros have upper –3 dB points below 100 Hz whereas the MHD sensors can have –3dB points above 1.5 kHz, set via the low pass filter in the internal electronics.

The first high pass filter corner in the MHD ARS is actually dominated by the physics of the sense channel. The back-EMF produced by the MHD effect coupled with the viscosity will cause the fluid in the sensor to 'catch up' with the sensor case when subjected to low frequency angular rotation. In addition to the sense element corner, a second high pass corner is placed in the internal electronics of most MHD ARS models to remove the offset bias after the first stage of amplification. As mentioned before, the upper –3dB LPF corner is set in the integral electronics and is not limited by the sense element below 5 kHz.

Low Frequency Compensation for MHD ARS Models with 2 HPF Poles:

The low frequency (<100 Hz) response of the ARS-01, ARS-03, ARS-04, ARS-09, or Dynacube™ can be represented as

$$H(s) = \frac{Ks^2}{(s + 2\pi f_1)(s + 2\pi f_2)}$$

where: K = angular rate scale factor
f_1 = sense element corner
f_2 = electronics high pass filter (HPF) corner

We will use the typical ARS-01 response as an example to illustrate how the compensation filter C(s) can be used to restore the low frequency content of the ARS-01 rate output. The first frequency corner f_1 for a typical ARS-01 is the physical corner of the sense element at about 0.25 Hz. The second lower corner, f_2, is the high pass filter set to 0.065 Hz within the internal signal conditioning electronics enclosed within the header of the ARS-01.

An upper low pass filter corner f_H, typically 1650 Hz, is also set within the integral electronics of the ARS-01 but has negligible effect below 100 Hz. Using pole cancellation, a compensation filter was designed to effectively reduce the low frequency corner for the ARS-01. The continuous compensation filter C(s) can be used to effectively move the HPF poles down in frequency to improve the low frequency response (FRF). The compensation filter can be represented as:

$$C(s) = \frac{(s + 2\pi f_1)(s + 2\pi f_2)}{(s + 2\pi f_3)(s + 2\pi f_4)}$$

where f_3 and f_4 are the new corners set lower than f_1 and f_2. The compensated FRF $H_C(s)$ for the sensor using the compensation filter C(s) becomes:

$$H_C(s) = H(s)C(s) = \frac{Ks^2}{(s + 2\pi f_3)(s + 2\pi f_4)}$$

where f_3 and f_4 are the new corner frequencies. The compensated response Hc(s) will behave as though it has lower frequency poles at f_3 and f_4 ($f_3 = f_4 = 0.002$ Hz for this example) which are lower than the original ARS-01 poles at f_1 and f_2 ($f_1 = 0.25$ Hz, $f_2 = 0.065$ Hz). Figure 2 is the typical normalized ARS-01 magnitude and phase response overlaid with the compensation filter response, C(s), and the extended (compensated) ARS-01 response, Hc(s)=H(s)C(s).

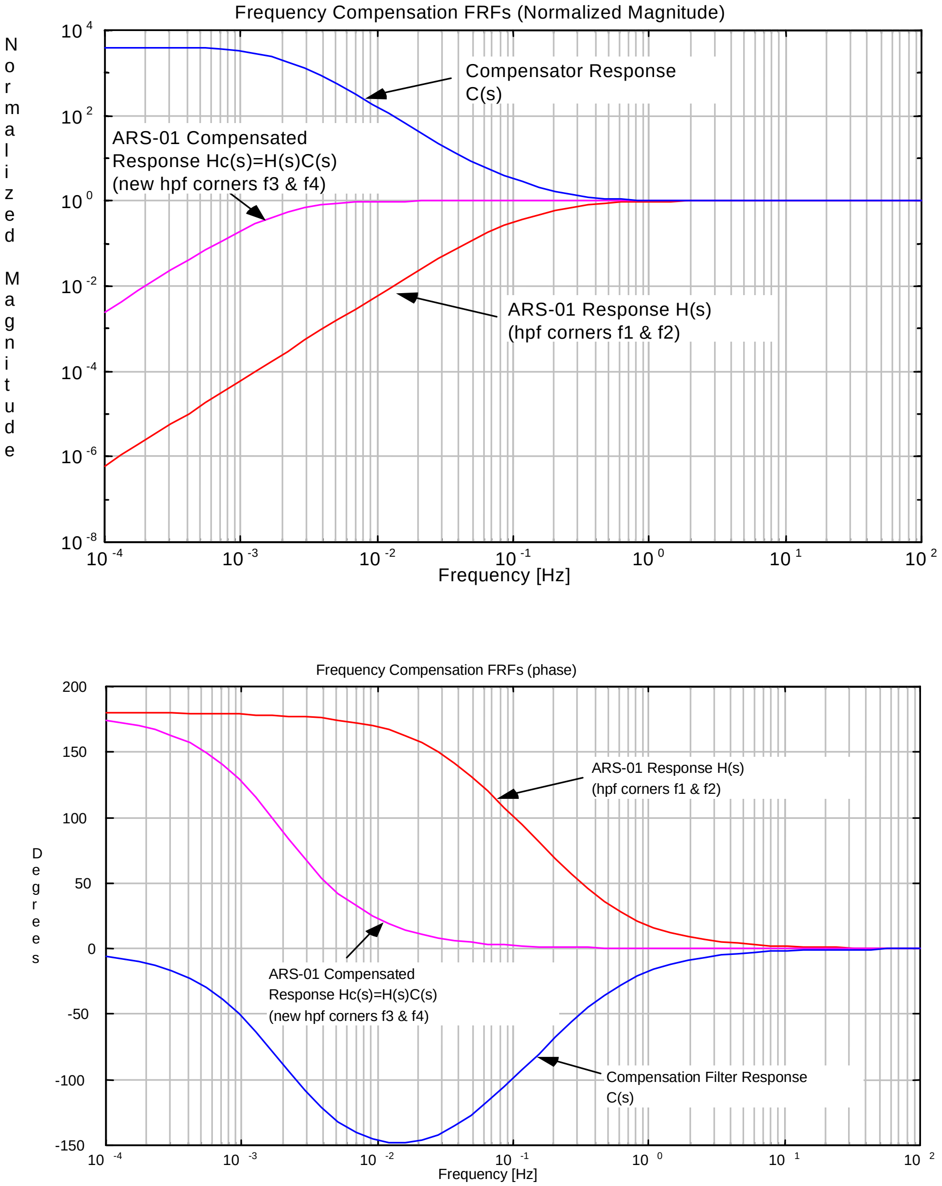

Figure 2. Overlay of the Normalized Response of the ARS-01, H(s), the compensation filter C(s), and the extended (compensated) ARS-01 Response, Hc(s)=H(s)C(s).

A z-domain digital filter can be used to compensate the rate signal time history to restore low frequency rate information, which can then be integrated to yield angle. The digital filter C(z) is found using zero-pole mapping technique.

$$C(z)=\frac{e^{-2\pi f_3\tau}e^{-2\pi f_4\tau}\left(z-e^{-2\pi f_1\tau}\right)\left(z-e^{-2\pi f_2\tau}\right)}{e^{-2\pi f_1\tau}e^{-2\pi f_2\tau}\left(z-e^{-2\pi f_3\tau}\right)\left(z-e^{-2\pi f_4\tau}\right)}$$

where τ is the sample period or 1/sampling frequency.

The difference equation for C(z) is then,

$$y_k = \left[\frac{e^{-2\pi f_3 \tau} e^{-2\pi f_4 \tau}}{e^{-2\pi f_1 \tau} e^{-2\pi f_2 \tau}}\right]\left[x_k - \left(e^{-2\pi f_1 \tau} + e^{-2\pi f_2 \tau}\right)x_{k-1} + e^{-2\pi f_1 \tau} e^{-2\pi f_2 \tau} x_{k-2}\right] + \left(e^{-2\pi f_3 \tau} + e^{-2\pi f_4 \tau}\right)y_{k-1} - e^{-2\pi f_3 \tau} e^{-2\pi f_4 \tau} y_{k-2}$$

where x_k = uncompensated rate input
y_k = compensated rate output.
τ = the time step between samples

Low Frequency Compensation for MHD ARS Models with 1 HPF Pole (ARS-06):
The low frequency (<100 Hz) response of the ARS-06 can be represented as

$$H(s) = \frac{Ks}{(s + 2\pi f_1)}$$

where: K = angular rate scale factor
f_1 = sense element corner

An upper low pass filter corner f_H, typically 1650 Hz, is also set within the integral electronics of the ARS-06 but has negligible effect below 100 Hz. The compensation filter for the single pole HPF MHD can be represented as:

$$C(s) = \frac{(s + 2\pi f_1)}{(s + 2\pi f_2)}$$

where f_2 is the new corner set lower than f_1. The compensated FRF $H_C(s)$ for the sensor using the compensation filter C(s) becomes:

$$H_C(s) = H(s)C(s) = \frac{Ks}{(s + 2\pi f_2)}$$

where f_2 is the new corner frequency. The compensated response Hc(s) will behave as though it has a lower frequency pole at f_2 rather than at f_1.

The difference equation for C(z) is then,

$$y_k = \left[\frac{e^{-2\pi f_2 \tau}}{e^{-2\pi f_1 \tau}}\right]\left[x_k - \left(e^{-2\pi f_1 \tau}\right)x_{k-1}\right] + e^{-2\pi f_2 \tau} y_{k-1}$$

where x_k = uncompensated rate input
y_k = compensated rate output.
τ = the time step between samples

Implementation of either digital filter is relatively simple on a computer. Appendix A provides an example of the digital filter implemented in MATLAB^{TM} that was used to compensate an ARS-01 with the sensitivity or scale factor, Kω=50mV/rad/sec, and f_1 and f_2 corners of 0.25 Hz and 0.065 Hz respectively. This program could be implemented for any MHD ARS using the measured scale factor Kω, and corner frequencies f_1 and f_2 that are supplied with the test data for each MHD ARS model. The extended corners f_3 and f_4 are set within the program. This particular program can also be implemented in any spreadsheet program if MATLAB™ is not available. The digital compensation algorithm provided is typically used for post-processing although the compensation filter could also be loaded into a Digital Signal Processor (DSP) for real time applications.

Summary

A digital compensation filter can be used to extend the low frequency response of the MHD ARS to yield accurate rate and angular measurements for time events lasting a few seconds, with best results for time histories of 1 second or less. Each application requires different rate or displacement accuracy based on time history length. Experimentation using the digital compensation filter by varying the extended corners is recommended to get an understanding of how the filter behaves. Several factors can effect the performance of the digital compensation filter, i.e. the accuracy of the A/D converters, the stability of the sample rate, and the methods used to remove pre-event biases and trends from the ARS-01 raw measurement data that cause errors when using the compensation filter.

MATLAB™ is a software product from The Math Works, Inc., South Natick, MA.

Appendix A: ARS-01 Compensation Example:

An actual example using real rate data measured with an ARS-01 is useful to illustrate how the compensation filter can be used to restore low frequency content. An optical encoder was used as the reference to measure the input angular displacement. The rate reference was calculated from the encoder displacement by differentiating the encoder reference angle data. Figure 3 overlays the input rate reference, (differentiated optical encoder angle) with the non-compensated ARS-01 rate output, and the compensated ARS-01 using the algorithm provided in Appendix A. Figure 4 shows the same overlays in displacement. The rate profile is basically in one direction starting from rest and then increasing to a peak angular rate of over 600 °/s and then back to zero rate again. The compensated ARS-01 shows very close agreement to the true input rate as compared to the non-compensated ARS-01 result. This example illustrates how effective the compensation filter can be in restoring the low frequency rate content of an MHD ARS for rate measurements lasting up to two seconds.

In many applications precision attitude or angular position versus time is needed. The compensated and non-compensated ARS-01 angular rates were integrated versus time to yield angular displacement and subsequently overlaid with the encoder reference angle for direct comparison as shown in Figure 4. Similar to the rate comparison, the non-compensated ARS-01 angular displacement result shows considerable error whereas the compensated ARS-01 result was in close agreement with the encoder reference angle. This example clearly illustrates the importance of using the compensation filter to compensate the angular rate prior to time integration of the MHD ARS to yield angular displacement. The compensation filter should be used for applications where angular position must be precisely known for up to a few seconds.

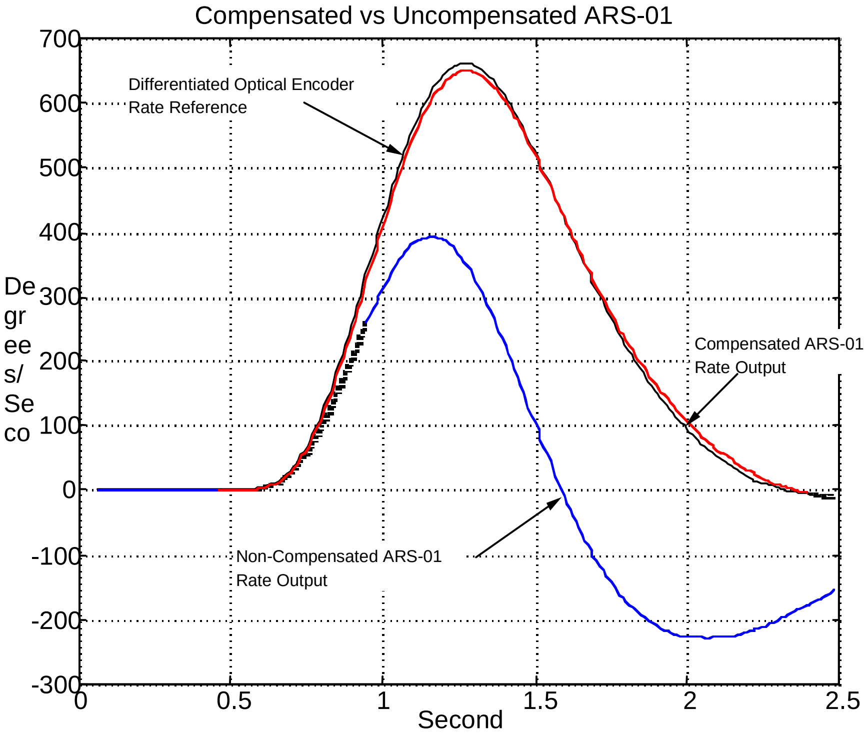

Figure 3. Overlays of the Input Rate Reference (differentiated encoder angle), the Non-Compensated ARS-01, and the Compensated ARS-01 Rate Results.

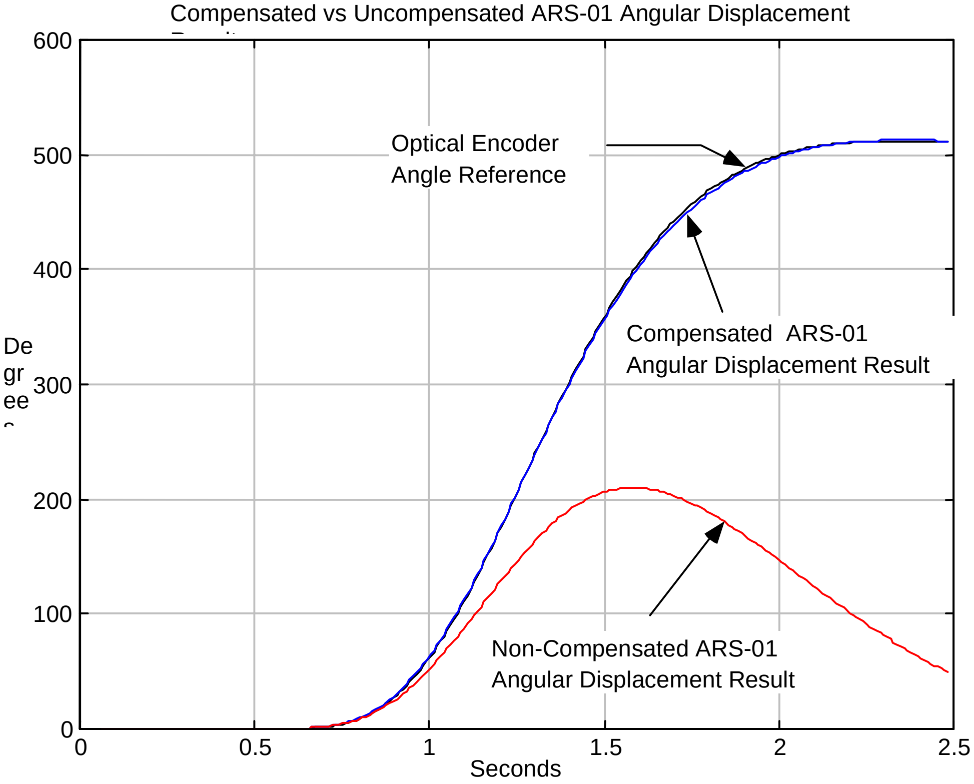

Figure 4. Overlays of the Optical Encoder Angle Reference, the Non-Compensated ARS-01 Result, and the Compensated ARS-01 Angular Displacement Result.

MATLAB^{TM} Digital Compensation Filter Algorithm

```
% Frequency Compensation Filter Example
% Load rate table encoder position and raw ARS data

%file ars_exam includes time (time), angle (ang), and raw MHD sensor (ars) data
load ars_exam

%calculate sample rate, assumes a fixed sample rate
tau=time(2)-time(1);
srate=1/tau;

%example ARS-01 scale factor in V/rad/s
Kwr=.050;

%converts scale factor into V/°/s
Kw=Kwr*pi/180;

%scale raw ARS-01 data by scale factor in V/°/s
ars=ars/Kw;

%Loop finds start of the impact using the encoder position data
i=1;
while ang(i) == 0,i=i+1;end
n_st=i-1
n=length(time);

%Calculate and remove the pre-impact bias from the scaled ARS-01 data
bias_ars=mean(ars(1:n_st));

%ars becomes the scaled ARS-01 data with pre-impact bias removed
ars=ars-bias_ars;

%hpf pole descriptions
f1=.25;  %hpf corner due to ARS-01 sense channel
f2=.065;   %hpf corner due to ARS-01 electronics
f3=.002;  %new compensated hpf corner 1
f4=.002;  %new compensated hpf corner 2

%Calculate digital filter coefficients based on hpf poles & sample period tau
a=exp((-1)*2*pi*f1*tau);
b=exp((-1)*2*pi*f2*tau);
c=exp((-1)*2*pi*f3*tau);
d=exp((-1)*2*pi*f4*tau);

%initialize output rate vector to zero
rate=zeros(n,1);

%factor for unity gain at z = 0, s = inf
fact=c*d/(a*b);
```

```
%initialize the 1st 2 samples of rate after the motion starts
rate(n_st)=ars(n_st);
rate(n_st+1)=ars(n_st+1);

% Apply digital compensation filter, rate(i) is the frequency compensated ARS-01 rate
for i=n_st+2:n;
rate(i)=fact*(ars(i)-(a+b)*ars(i-1)+a*b*ars(i-2))+(c+d)*rate(i-1)-c*d*rate(i-2);
end;

%Overlay compensated vs uncompensated ARS-01 rates
plot(time,rate,'-',time,ars,'--');
title('Compensated vs Uncompensated ARS-01 Angular Rates'),
xlabel('Seconds'),
ylabel('Degrees/Second');
grid
pause

% Integrate compensated and uncompensated rates to obtain angular displacement
%initialize compensated output angle vector to zero
ang_comp =zeros(n,1);

%initialize uncompensated output angle vector to zero
ang_noncomp =zeros(n,1);

%sum the rate and ars samples
for i=2:n;
ang_comp(i)=rate(i)+ang_comp(i-1);
ang_noncomp(i)=ars(i)+ang_noncomp(i-1);
end

%divide by the sample rate to obtain the integrated angle
ang_comp=ang_comp/srate;
ang_noncomp=ang_noncomp/srate;

% Overlay Encoder Angle, Compensated and Uncompensated ARS-01 results
plot(time,ang,'-',time,ang_comp,'--',time,ang_noncomp,'-');
title('Compensated vs Uncompensated ARS-01 Angular Displacement'),
xlabel('Seconds'),
ylabel('Degrees');
grid
break
```